NOTATION

λ, α, w_λ, length, amplitude, and velocity of motion of a wave; ρ_ℓ, μ_ℓ, σ, density, coefficient of dynamic viscosity, and specific surface energy of the liquid that is breaking up; k_λ, proportionality factor in the expression $\lambda = k_\lambda \lambda_{min}$; D, diameter of the liquid drops and jets breaking up; $We \equiv \rho_{amb} v^2 D/\sigma$, Weber number; $We_{re} \equiv \rho_{amb} v_{re}^2 D/\sigma$, reduced Weber number; $L_p \equiv \frac{\rho_\ell D \sigma}{\mu_\ell^2}$, Laplace number; ρ_{amb}, density of the oncoming gas stream; v, relative velocity of the oncoming stream; τ, time; S, path length traveled by the wave; t, characteristic size of the volume breaking up; θ, angle between the point under consideration and the frontal point on the surface volume breaking up in the polar coordinate system. Indices: ℓ, liquid; amb, ambient medium; gr, growth, mo, motion; cr, critical; min, minimum; 1, 2) first and second breakup conditions; re, reduced.

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ANALYSIS OF SELF-SIMILAR LAMINAR FLOWS IN SLOT CHANNELS WITH ONE PERMEABLE WALL

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UDC 582.542.2:532.546

An analysis is made of the fluid flows in a plane slot permeable channel. It is shown that for large numbers R (suction) self-similar solutions exist.

Plane and axisymmetric laminar flows are observed in many modern engineering elements [1, 2]. These are systems of "porous" effusion cooling, heat pipes, heat exchange sublimation apparatus, apparatus for thermostatic regulation of large-scale objects [3, 4], and distributive collectors of heat exchangers [5]. It is difficult to assume a uniform discharge distribution of the heat carrier in channels without analyzing the flows in such apparatus.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 3, pp. 432-436, September, 1985. Original article submitted April 2, 1984.

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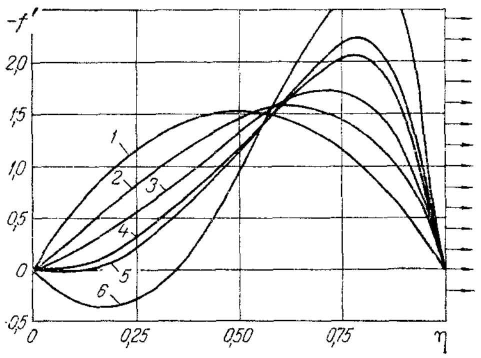

Fig. 1. Profiles of the function $f'(\eta)$ proportional to the longitudinal velocity component u in a plane channel with unilateral suction of different intensity: 1) $R_{10} = 1.5$; 2) 6.3; 3) 12.4; 4) 13.2; 5) 13.8; 6) 14.5.

A computational analysis of possible self-similar laminar flows in porous pipes for homogeneous blowing and suction characterized by different values of the number R is executed in [6], where it was shown that for both suction ($R > 0$) and blowing ($R < 0$) several solutions of the problem exist. A range of numbers R was here set up for which stationary self-similar flows do not exist. Plane and axisymmetric self-similar flows that occur for unilateral blowing in a slot channel were studied in [1, 7]. The qualitative analysis of the solution executed for large numbers R does not yield practical information about flows with unilateral suction.

To obtain a complete portrait of the possible self-similar stationary flows in slot channels with one permeable wall it is necessary to investigate plane and axisymmetric flows due to the suction of substance through one of the channel walls.

As is known [8, 9], the longitudinal u and transverse w velocity components of a self-similar flow in a slot channel can be represented in the form

$$u = -\frac{1}{m}\frac{x}{H}|W|f'(\eta), \quad w = |W|f(\eta). \tag{1}$$

The function $f(\eta)$ is a solution of the boundary-value problem for the differential equation

$$f''' - R\left(ff'' - \frac{1}{m}f'^2\right) = k. \tag{2}$$

Here the parameter m = 1 and m = 2 in the plane and axisymmetric problems, x, z are the longitudinal and transverse coordinates, $\eta = z/H$; H is the height of the slot channel, w is the transverse velocity components on the permeable wall z = H (positive for suction), and $R = wH/\nu$ is the Reynolds number.

The boundary conditions are

$$f(0) = 0, \quad f'(0) = 0, \quad f(1) = \mathrm{sign}\, w, \quad f'(1) = 0. \tag{3}$$

The study of all possible self-similar flows in a slot channel reduces to a one-parameter analysis (the pressure gradient parameter k is kept in mind here) for whose numerical realization it is expedient to replace the nonlinear boundary-value problem (2), (3) by the Cauchy problem

$$F''' + \frac{1}{m}F'^2 - FF'' = K, \tag{4}$$

$$F(0) = 0, \quad F'(0) = 0, \quad F''(0) = (-1)^n. \tag{5}$$

The primes here denote differentiation with respect to the new dimensionless variable $\eta_1 = b\eta$, and $F(\eta_1) = A^{-1}f(\eta)$. It is assumed that

$$\frac{b}{A} = R, \quad b^2A = \pm f''(0), \quad \frac{k}{b^3A} = K. \tag{6}$$

To analyze flows due to the injection of fluid through a permeable wall, it is necessary to take n = 1, while K is variated within the limits of the positive semiaxis $(0, +\infty)$. Profiles of the longitudinal velocity component u, proportional to $f(\eta)$, with an inflection point correspond to positive values of the parameter K during suction in a plane slot channel.

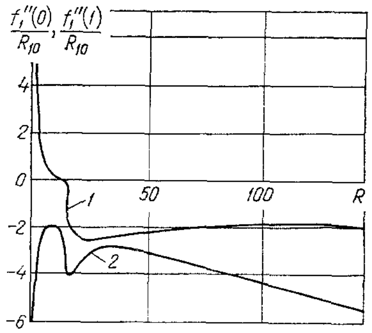

Fig. 2

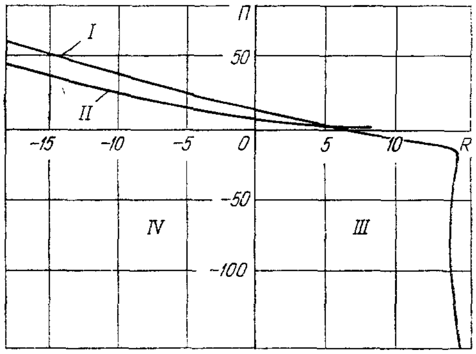

Fig. 3

Fig. 2. Change in the profile of the function f"(1) as a function of the suction intensity: 1) $f''_1(0)/R_{10}$; 2) $f''_1(1)/R_{10}$.

Fig. 3. Dependence of the pressure gradient parameter Π on the suction intensity: I) Plane problem; II) axisymmetric problem; III) suction; IV) blowing.

Among such regimes are flows with a recirculation zone abutting the impermeable wall. To find such flows n = 0 is given. An analogous assumption is also made in the remaining cases.

For n = 0 and R > 0 (suction), exactly as for n = 1 and R < 0 (blowing), Eq. (4) is integrated to the second zero $\eta_1 = \eta_{12}$ and for n = 1 to the third zero $\eta_1 = \eta_{13}$ of the functions $F'(\eta_1)$ $F'(\eta_{1j}) = 0$; $\eta_{11} = 0$ (the upper limit of integration η_{1j} corresponds to the permeable wall). The Reynolds number R characterizing the suction (blowing) intensity and the pressure gradient parameter $\Pi = H^3 \partial P/\partial x/\rho \nu w x$ (ρ is the fluid density) are calculated from the formulas

$$R = \eta_{1j} F(\eta_{1j}), \quad \Pi = \frac{K\eta_{1j}^2}{mF(\eta_{1j})}.$$

Profiles of the function $f'(\eta)$ are shown in Fig. 1; for $R \downarrow 0$ these profiles degenerate into a parabola symmetric with respect to the middle plane of the channel (η = 0.5), i.e., as a Poisseuille longitudinal velocity distribution is realized for low-intensity blowing ($R \uparrow 0$), here $\Pi \uparrow 12$. As the number R grows, the maximum of the longitudinal velocity component shifts towards the permeable wall, which is accompanied by an increase in the friction stress proportional to the function $f^{II}(1)$ (see Fig. 2). The friction stress on the impermeable wall $f^{II}(0)$ decreases with the growth of the number R and for $R \uparrow R^I \approx 6.3$ the curvature of the profile of the longitudinal velocity component tends to zero near the impermeable wall. A zero value of the parameters K and Π corresponds to this regime. Consequently, by virtue of (4) and (5) even in that case $f^{III}(0) = 0$. Therefore, the passage through $R = R^I$ is accompanied by the appearance of an inflection point on the profile $f^I(\eta)$, which moves from the impermeable wall to the middle plane as R grows.

For $R \uparrow R^{II} \approx 13$ the function proportional to the friction stress is $f^{II}(0) \downarrow 0$ and $F^{II}(0) < 0$ for $R > R^{II}$, i.e., the appropriate solutions describe flows with a reverse flow zone whose width increases as the number R grows (see Fig. 1).

The dependence of the parameter Π on the number R is shown in Fig. 3. For $R \lesssim 14$ it is practically linear and is a smooth continuation of the corresponding dependence for unilateral blowing (R < 0). For $-\infty < R < R^I$ $\Pi > 0$, i.e., a pressure drop in the direction of the main stream corresponds to these regimes. For $R > R^I$ in a plane channel, the pressure growth due to a diminution in velocity ($\partial u/\partial x < 0$) predominates over its drop because of the viscous dissipation.

In the range $R^{III} < R < R^{IV}$ ($R^{III} \approx 14.1$; $R^{IV} \approx 14.6$), three solutions correspond to each value of R and can generally be characterized by substantially different parameters Π.

For $R > R^{IV}$ the modulus of the parameter Π (Fig. 3) grows monotonically to the right of the mentioned interval while the solutions $f^I(\eta)$ approximate half a sinusoid in shape as $R \uparrow \infty$, i.e., for large numbers R fully recirculation flows will be observed in a plane channel according to the self-similar solutions considered, and all the fluid being sucked from the channel should flow only in a thin layer along the permeable wall.

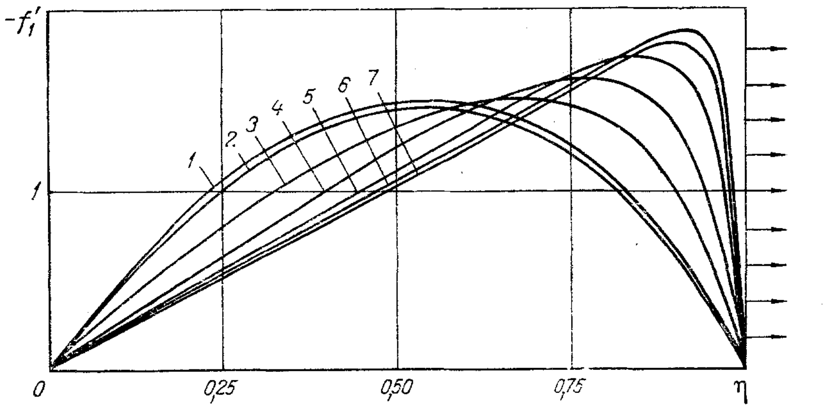

Fig. 4. Profiles of the longitudinal velocity component in the gap between discs for unilateral suction of different intensity: 1) R = 0.9; 2) 1.3; 3) 4.3; 4) 7.8; 5) 13.2; 6) 21.6; 7) 29.4.

It is known from the theory of hydrodynamic stability of parallel flows that the presence of an inflection point in the velocity profile is ordinarily the criterion for laminar flow instability. Consequently, it can be expected that flows corresponding to the self-similar solutions considered are unstable for $R > R^{II}$ although, as is shown in [10], the presence of a transverse velocity can noticeably deform the boundaries of the stable regimes found from the solution of the Orr–Sommerfeld equation for a parallel flow with the same distribution of the longitudinal velocity component. Moreover, it should be taken into account that for the physical realization of self-similar flows for large positive numbers R, it is necessary to assure an appropriate velocity distribution at the input to the plane channel since flow stabilization occurs on a section of length $\ell_0 \approx 0.04$ ReH (this is confirmed in [11] by a numerical experiment for a channel with a permeable wall), where Re is the Reynolds number constructed with the mean longitudinal velocity component with respect to the section $\langle u \rangle = \int_0^H u dz$, while the characteristic dimension ℓ is the distance between the entrance to the channel and the section under consideration. Therefore $\ell_0/\ell \approx 0.04$ R, i.e., for sufficiently large numbers R the lengths ℓ_0 and ℓ are commensurate. Experiments [12] performed on a porous pipe with suction showed that turbulent fluctuations occur near the wall almost simultaneously with the appearance of the reverse flows.

The axisymmetric flow pattern in a slot channel for unilateral suction and blowing is substantially simpler than that described above. The longitudinal (radial) velocity component profiles in the gap between discs with unilateral suction of different intensities are shown in Fig. 4. For $R \downarrow 0$ these profiles degenerate, in the plane case, into a parabola symmetric with respect to the plane $\eta = 0.5$, and the parameter $\Pi \uparrow 6$. The increase in R from 0 to ∞ is accompanied by monotonic deformation of the parabolic profile $f^I(\eta)$ into a triangular with vertices at the points $\eta = 0$ $f^I(0) = 0$ and $\eta = 1$ $f^I(1) = 2$ on the impermeable and permeable walls, respectively. The parameter Π (Fig. 4) here decreases monotonically to zero. In the axisymmetric case the hydraulic losses always predominate over the pressure rise due to a diminution in the velocity (Π > 0).

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PROPAGATION OF A PLANAR SHOCK THERMAL WAVE IN A NONLINEAR MEDIUM

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UDC 536.2.01

Conditions are analyzed for a strong discontinuity of a thermal field in a nonlinear medium, possessing thermal relaxation. The solution of the heat-transfer equations is obtained in the region between the mobile boundaries and the front of the strong discontinuity.

1. Analysis of Strong Discontinuity Conditions of a Thermal Field. The heat-transfer equations in a medium with a relaxing thermal flow [1, 2] are written in the following form for one-dimensional processes with planar symmetry

$$\rho h_t + q_x = 0, \quad \rho, \ \gamma - \text{const}, \tag{1}$$

$$L_x + \gamma q_t + q = 0, \quad h = \int_0^T c_p(T)\, dT, \quad L = \int_0^T \lambda(T)\, dT. \tag{2}$$

Shock thermal waves can be generated in nonlinear media possessing thermal relaxation [2, 3]. In particular, an important object of application of the heat-transfer model (1), (2) are thermal perturbations in liquid helium [4, 5]. It is well known that second sound shock waves can occur in liquid helium at temperature 1.2K < T < 2.0K; the physical analysis of this effect and a bibliography are given in [5, 6]. In the presence of relaxation properties of the medium surfaces of strong discontinuity are also formed in other physicomechanical processes, for example, in liquid filtration [7]. This question is discussed in [2].

To obtain conditions of dynamic compatibility at the strong discontinuity line of the thermal field the energy conservation law must be selected in integral form, and then the method of [8, 9] must be applied:

$$N\{\rho h\} = \{q\}, \quad N = dx_j/dt. \tag{3}$$

Here the brackets denote the jump of the corresponding functions during transition through the strong continuity line $x = x_j(t)$.

Also possible are statements of the heat-transfer problem, in which the single condition (3) is insufficient to guarantee uniqueness of the solution, and, according to [9], an additional relation is required at the discontinuity.

Homel Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 3, pp. 436-443, September, 1985. Original article submitted May 24, 1984.

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